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14. ABSTRACT

Stability properties of hydromagnetic waves (shear and compressional Alfven waves) in spatially homogeneous plasma are investigated when the equilibrium particle velocity distributions in both parallel and perpendicular directions (in reference to the ambient magnetic field) are modeled by kappa distributions. Analysis is presented for the limiting cases $|\xi_\alpha| \ll 1$ and $|\xi_\alpha| \gg 1$ for which solutions of the dispersion relations are analytically tractable. Here $\xi_\alpha(\alpha=e,i)$ is the ratio of the wave phase speed and the electron (ion) thermal speed. Both low and high β (=plasma pressure/magnetic pressure) plasmas are considered. The distinguishing features of the hydromagnetic waves in kappa distribution plasma are (1) both Landau damping and transit-time damping rates are larger than those in Maxwellian plasma because of the enhanced high-energy tail of the kappa distribution and (2) density and temperature perturbations in response to the electromagnetic perturbations are different from those in Maxwellian plasma when $|\xi_\alpha| \ll 1$. Moreover, frequency of the oscillatory stable modes (e.g., kinetic shear Alfven wave) and excitation condition of the nonoscillatory (zero frequency) unstable modes (e.g., mirror instability) in kappa distribution plasma are also different from those in Maxwellian plasma. Quantitative estimates of the differences depend on the specific choice of the kappa distribution. For simplicity of notations, same spectral indices $\kappa_\parallel$ and $\kappa_\perp$ have been assumed for both electron and ion population. However, the analysis can be easily generalized to allow for different values of the spectral indices for the two charged populations.

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Hydromagnetic waves and instabilities in kappa distribution plasma

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Stability properties of hydromagnetic waves (shear and compressional Alfven waves) in spatially homogeneous plasma are investigated when the equilibrium particle velocity distributions in both parallel and perpendicular directions (in reference to the ambient magnetic field) are modeled by kappa distributions. Analysis is presented for the limiting cases $|\xi_\alpha| \ll 1$ and $|\xi_\alpha| \gg 1$ for which solutions of the dispersion relations are analytically tractable. Here $\xi_\alpha(\alpha=e,i)$ is the ratio of the wave phase speed and the electron (ion) thermal speed. Both low and high β (=plasma pressure/magnetic pressure) plasmas are considered. The distinguishing features of the hydromagnetic waves in kappa distribution plasma are (1) both Landau damping and transit-time damping rates are larger than those in Maxwellian plasma because of the enhanced high-energy tail of the kappa distribution and (2) density and temperature perturbations in response to the electromagnetic perturbations are different from those in Maxwellian plasma when $|\xi_\alpha| \ll 1$. Moreover, frequency of the oscillatory stable modes (e.g., kinetic shear Alfven wave) and excitation condition of the nonoscillatory (zero frequency) unstable modes (e.g., mirror instability) in kappa distribution plasma are also different from those in Maxwellian plasma. Quantitative estimates of the differences depend on the specific choice of the kappa distribution. For simplicity of notations, same spectral indices $\kappa_\parallel$ and $\kappa_\perp$ have been assumed for both electron and ion population. However, the analysis can be easily generalized to allow for different values of the spectral indices for the two charged populations.
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I. INTRODUCTION

Low frequency (lower than the ion cyclotron frequency) and long perpendicular wavelength (longer than the ion gyroradius) electromagnetic waves, often referred to as hydromagnetic waves, are often observed and/or invoked to explain the phenomena in both space and laboratory plasmas. These waves and their stability properties in Maxwellian plasma have been investigated quite extensively for many years by many authors and a good discussion on them can be found in the plasma textbook by Stix.[1] In collisionless plasma, however, particle velocity distributions can often depart from being Maxwellian. For example, in naturally occurring plasma such as plasma in the planetary magnetospheres and in the solar wind, the particle velocity distributions are observed to have non-Maxwellian (power-law), high-energy tail.[2] The distribution function that can better model such particle velocity distributions is known as the generalized Lorentzian or the kappa distribution[3] with functional dependence of the form $f_0(v) \sim [1+v^2/(\kappa\theta^2)]^{-(\kappa+1)}$. For finite values of the spectral index κ, the kappa distribution has power-law tail at velocities larger than the thermal velocity θ, and it approaches a Maxwellian distribution $[\sim\exp(-v^2/\theta^2)]$ in the limit as $\kappa\to\infty$. Typical values of κ for space plasmas are in the range of 2–6. In the last several years, many authors have studied electrostatic and electromagnetic waves in spatially homogeneous plasma using different types of kappa distributions for the equilibrium state.[4–15] In a recent paper,[16] we studied low frequency (lower than the ion cyclotron frequency) and long perpendicular wavelength (longer than the ion gyroradius) electrostatic waves in spatially inhomogeneous, current-carrying, anisotropic plasma, where the equilibrium particle velocity distributions were modeled by different kappa distributions. In the present paper, we investigate the stability properties of the hydromagnetic waves in spatially homogeneous plasma, where the equilibrium particle velocity distributions in both parallel and perpendicular directions with respect to the ambient magnetic field are modeled by kappa distributions.

The paper is organized in the following way. In Sec. II, we describe the general mathematical formalism leading to the derivation of the dispersion relations for hydromagnetic waves in kappa distribution plasma. In Sec. III, we analyze the dispersion relation in various limits and describe the stability properties of the well-known hydromagnetic waves. Both low-β ($\beta \ll 1$) (β=plasma pressure/magnetic pressure) and high-β ($\beta>1$) plasmas are considered. In Sec. IV, we discuss the distinguishing features of the hydromagnetic waves in kappa distribution plasma and offer some physical interpretations.

II. MATHEMATICAL FORMALISM

We adopt a Cartesian coordinate system whose z-axis is along the ambient uniform magnetic field $\mathbf{B}_0$ and consider small amplitude electromagnetic perturbations represented by $(\tilde{\mathbf{E}},\tilde{\mathbf{B}})\exp(-i\omega t+ik_\perp y+ik_\parallel z)$ with $k_\parallel>0$ and $k_\perp>0$. Electromagnetic modes in plasma can be described in terms of any three of the six field variables $(\tilde{\mathbf{E}},\tilde{\mathbf{B}})$ by eliminating the other three with the help of $\mathbf{k}\times\tilde{\mathbf{E}}=(\omega/c)\tilde{\mathbf{B}}$ and $\mathbf{k}\cdot\tilde{\mathbf{B}}=0$. The three field variables that we consider to be physically most meaningful for hydromagnetic waves are $\tilde{E}_z$, $\tilde{B}_x$, and $\tilde{B}_z$. The

magnitude of $\tilde{E}_z$ is a measure of the non-MHD (magnetohydrodynamic) character of the waves and $\tilde{B}_z$ corresponds to the compressional component of the magnetic perturbation, which characterizes waves in finite-β plasma. Since $\omega \ll ck$ for hydromagnetic waves, where $k^2=k_\parallel^2+k_\perp^2$, the displacement current can be neglected and, consequently, $\nabla \cdot \tilde{\mathbf{j}}=0$ (implying quasineutrality) can be assumed. The three equations that determine $\tilde{E}_z$, $\tilde{B}_x$, and $\tilde{B}_z$ are then the quasineutrality condition

$$\sum_\alpha q_\alpha \tilde{n}_\alpha \equiv \sum_\alpha q_\alpha \int d\mathbf{v} \tilde{f}_\alpha = 0 \tag{1}$$

and the perpendicular (to $\mathbf{B}_0$) components of Ampere's law given by

$$\tilde{B}_z = -\frac{4\pi i k_\perp}{ck^2} \tilde{j}_x \equiv -\frac{4\pi i k_\perp}{ck^2} \sum_\alpha q_\alpha \int d\mathbf{v} v_x \tilde{f}_\alpha, \tag{2}$$

$$\tilde{B}_x = -\frac{4\pi i}{ck_\parallel} \tilde{j}_y \equiv -\frac{4\pi i}{ck_\parallel} \sum_\alpha q_\alpha \int d\mathbf{v} v_y \tilde{f}_\alpha. \tag{3}$$

Here q_α is the charge and $\tilde{n}_\alpha$ is the perturbed density of the charged particle species $\alpha(=e,i)$, and $\tilde{j}$ are the components of the perturbed current density. The perturbed quantities are calculated from the perturbed particle distribution function $\tilde{f}_\alpha \equiv \tilde{f}_\alpha(\mathbf{k},\mathbf{v},\omega)$, as shown in Eqs. (1)–(3).

We refer to a cylindrical coordinate system in velocity space with its z-axis parallel to $\mathbf{B}_0$, so that $v_x = v_\perp \cos\varphi$, $v_y = v_\perp \sin\varphi$, and $v_z \equiv \mathbf{v}\cdot\mathbf{B}_0/B_0 = \mathrm{v}_\parallel$, where φ is the azimuth angle. The equilibrium distribution function $f_{\alpha 0}$, which can be constructed from the constants of motion ($v_\perp^2$ and $v_\parallel$) of the charged particle species α, is taken to be described by the product bi-Lorentzian-type kappa distribution function such as[2,16]

$$f_{\alpha 0}(v_\perp^2, v_\parallel) = \frac{n_0}{\pi^{3/2}\theta_{\alpha\perp}^2 \theta_{\alpha\parallel}} \frac{\Gamma(\kappa_\parallel+1)}{\kappa_\parallel^{1/2}\Gamma(\kappa_\parallel+1/2)} \left(1+\frac{v_\perp^2}{\kappa_\perp \theta_{\alpha\perp}^2}\right)^{-(\kappa_\perp+1)} \times \left(1+\frac{v_\parallel^2}{\kappa_\parallel \theta_{\alpha\parallel}^2}\right)^{-(\kappa_\parallel+1)}. \tag{4}$$

The function $f_{\alpha 0}(v_\perp^2, v_\parallel)$ is normalized to the equilibrium (unperturbed) particle density n_0, while $\theta_{\alpha\parallel}$ and $\theta_{\alpha\perp}$ are the parallel and perpendicular thermal speeds, respectively. The parallel and perpendicular pressures in the equilibrium state $p_{\alpha\parallel}$ and $p_{\alpha\perp}$ are

$$p_{\alpha\parallel} \equiv n_0 T_{\alpha\parallel} = m_\alpha \int d\mathbf{v} v_\parallel^2 f_{\alpha 0} = \left(\frac{2\kappa_\parallel}{2\kappa_\parallel - 1}\right)\frac{1}{2} m_\alpha n_0 \theta_{\alpha\parallel}^2, \tag{5}$$

$$p_{\alpha\perp} \equiv n_0 T_{\alpha\perp} = \frac{1}{2} m_\alpha \int d\mathbf{v} v_\perp^2 f_{\alpha 0} = \left(\frac{\kappa_\perp}{\kappa_\perp - 1}\right)\frac{1}{2} m_\alpha n_0 \theta_{\alpha\perp}^2 \tag{6}$$

for $\kappa_\parallel > 1/2$ and $\kappa_\perp > 1$. Here $T_{\alpha\parallel}$ and $T_{\alpha\perp}$ are the parallel and perpendicular effective particle temperatures, respectively. Two other parameters related to $p_{\alpha\parallel}$ and $p_{\alpha\perp}$ which naturally occur in the study of electromagnetic waves are $\beta_{\alpha\parallel} \equiv 8\pi p_{\alpha\parallel}/B_0^2 = [2\kappa_\parallel/(2\kappa_\parallel - 1)](4\pi m_\alpha n_0 \theta_{\alpha\parallel}^2/B_0^2)$ and $\beta_{\alpha\perp} \equiv 8\pi p_{\alpha\perp}/B_0^2 = [\kappa_\perp/(\kappa_\perp - 1)](4\pi m_\alpha n_0 \theta_{\alpha\perp}^2/B_0^2)$.

In our choice of the equilibrium distribution function we have allowed the possibility of different values of the spectral index in parallel and perpendicular directions. In fact, it may be reasonably argued that $\kappa_\perp > \kappa_\parallel$ because of some equilibration and isotropization in the perpendicular plane while preferential acceleration along the ambient magnetic field. Realistically, the spectral indices would also be different for the electron and the ion populations. But, for simplicity of notations, we have assumed same spectral indices for both the populations. However, the analysis presented in this section and in Secs. III and IV can be easily generalized to allow for different values of the spectral indices for the two populations.

Solving the linearized Vlasov equation by standard procedure (integration along the unperturbed particle orbits in the ambient magnetic field $\mathbf{B}_0$), we first obtain[17]

$$\tilde{f}_\alpha(\mathbf{k},\mathbf{v},\omega) = -\frac{q_\alpha}{m_\alpha k_\parallel} \sum_{n=-\infty}^{+\infty} i^{n+1} \frac{\exp(-i\mu_\alpha \cos\varphi + in\varphi)}{\omega - k_\parallel v_\parallel + n\Omega_\alpha} \left\{ \tilde{E}_z J_n(\mu_\alpha)\left(k_\parallel \frac{\partial}{\partial v_\parallel} - 2n\Omega_\alpha \frac{\partial}{\partial v_\perp^2}\right) + \left[\frac{n\Omega_\alpha}{ck_\perp} J_n(\mu_\alpha)\tilde{B}_x + i\frac{k_\parallel}{k_\perp}\frac{\mathrm{v}_\perp}{c} J_n'(\mu_\alpha)\tilde{B}_z\right] \times \left[k_\parallel \frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]\right\} f_{\alpha 0}(v_\perp^2, v_\parallel), \tag{7}$$

where $\Omega_\alpha = q_\alpha B_0/(m_\alpha c)$, $J_n(\mu_\alpha)$ is the Bessel function of the first kind, and the prime notation on J_n denotes its first derivative with respect to the argument $\mu_\alpha = k_\perp v_\perp/\Omega_\alpha$. As mentioned earlier, $\tilde{E}_x$, $\tilde{E}_y$, and $\tilde{B}_y$ have been eliminated in favor of $\tilde{E}_z$, $\tilde{B}_x$, and $\tilde{B}_z$. Using the identity

$$\exp(-i\mu_\alpha \cos\varphi) = \sum_{p=-\infty}^{+\infty} (-i)^p J_p(\mu_\alpha)\exp(-ip\varphi) \tag{8}$$

in Eq. (7) and carrying out the φ-integrations we then obtain

$$\int_0^{2\pi} d\phi \tilde{f}_\alpha = -\frac{2\pi i q_\alpha}{m_\alpha k_\parallel}\sum_{n=-\infty}^{+\infty}\frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\left\{\tilde{E}_z J_n^2\left(k_\parallel\frac{\partial}{\partial v_\parallel} - 2n\Omega_\alpha\frac{\partial}{\partial v_\perp^2}\right) + \left[\frac{n\Omega_\alpha}{ck_\perp}J_n^2\tilde{B}_x + i\frac{k_\parallel}{k_\perp}\frac{v_\perp}{c}J_nJ_n'\tilde{B}_z\right]\right.$$
$$\left.\times\left[k_\parallel\frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]\right\}f_{\alpha 0}(v_\perp^2, v_\parallel), \tag{9}$$

$$\int_0^{2\pi} d\varphi \cos\varphi \tilde{f}_\alpha = \frac{2\pi q_\alpha}{m_\alpha k_\parallel}\sum_{n=-\infty}^{+\infty}\frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\left\{\tilde{E}_z J_nJ_n'\left(k_\parallel\frac{\partial}{\partial v_\parallel} - 2n\Omega_\alpha\frac{\partial}{\partial v_\perp^2}\right) + \left[\frac{n\Omega_\alpha}{ck_\perp}J_nJ_n'\tilde{B}_x + i\frac{k_\parallel}{k_\perp}\frac{v_\perp}{c}(J_n')^2\tilde{B}_z\right]\right.$$
$$\left.\times\left[k_\parallel\frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]\right\}f_{\alpha 0}(v_\perp^2, v_\parallel), \tag{10}$$

$$\int_0^{2\pi} d\varphi \sin\varphi \tilde{f}_\alpha = \frac{2\pi i q_\alpha}{m_\alpha k_\parallel}\sum_{n=-\infty}^{+\infty}\frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\frac{n\Omega_\alpha}{k_\perp v_\perp}\left\{\tilde{E}_z J_n^2\left(k_\parallel\frac{\partial}{\partial v_\parallel} - 2n\Omega_\alpha\frac{\partial}{\partial v_\perp^2}\right) + \left[\frac{n\Omega_\alpha}{ck_\perp}J_n^2\tilde{B}_x + i\frac{k_\parallel}{k_\perp}\frac{v_\perp}{c}J_nJ_n'\tilde{B}_z\right]\right.$$
$$\left.\times\left[k_\parallel\frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]\right\}f_{\alpha 0}(v_\perp^2, v_\parallel), \tag{11}$$

which are relevant for the calculation of $\tilde{n}_\alpha$, $\tilde{j}_x$, and $\tilde{j}_y$. However, for the description of the low frequency and long parallel wavelength modes $[(\omega, k_\parallel \theta_{\alpha\parallel}) \ll \Omega_\alpha]$ considered here, Eqs. (9)–(11) can be simplified by taking $(\omega - k_\parallel v_\parallel + n\Omega_\alpha)^{-1} \cong (n\Omega_\alpha)^{-1}[1-(\omega - k_\parallel v_\parallel)/n\Omega_\alpha]$ for $n\neq 0$ in the sum and using the identities $\Sigma_{n\neq 0}J_n^2 = 1 - J_0^2$, $\Sigma_{n\neq 0}J_nJ_n' = -J_0J_0'$, $\Sigma_{n\neq 0}nJ_n^2 = 0$, $\Sigma_{n\neq 0}J_n^2/n = 0$, $\Sigma_{n\neq 0}J_nJ_n'/n = 0$, and $\Sigma_{n\neq 0}(J_n')^2/n = 0$. Keeping only the leading order nonzero terms in Eqs. (9)–(11), we then find

$$\int_0^{2\pi} d\varphi \tilde{f}_\alpha = \frac{2\pi i q_\alpha}{m_\alpha k_\parallel}\left\{\tilde{E}_z\left[2(1-J_0^2)\frac{\partial}{\partial v_\perp^2} - \frac{J_0^2}{\omega - k_\parallel v_\parallel}k_\parallel\frac{\partial}{\partial v_\parallel}\right] - \left[(1-J_0^2)\frac{\tilde{B}_x}{ck_\perp} + i\frac{k_\parallel}{k_\perp}\frac{v_\perp}{c}\frac{J_0J_0'}{\omega - k_\parallel v_\parallel}\tilde{B}_z\right]\right.$$
$$\left.\times\left[k_\parallel\frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]\right\}f_{\alpha 0}(v_\perp^2, v_\parallel), \tag{12}$$

$$\int_0^{2\pi} d\varphi \cos\varphi \tilde{f}_\alpha = \frac{2\pi q_\alpha}{m_\alpha k_\parallel}\left\{\left(\frac{\tilde{E}_z}{\omega - k_\parallel v_\parallel} - \frac{\tilde{B}_x}{ck_\perp}\right)J_0J_0' + i\frac{k_\parallel}{k_\perp}\frac{v_\perp}{c}\tilde{B}_z\left[\frac{(J_0')^2}{\omega - k_\parallel v_\parallel} - 2(J_1')^2\frac{\omega - k_\parallel v_\parallel}{\Omega_\alpha^2}\right]\right\}$$
$$\times\left[k_\parallel\frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]f_{\alpha 0}(v_\perp^2, v_\parallel), \tag{13}$$

$$\int_0^{2\pi} d\varphi \sin\varphi \tilde{f}_\alpha = \frac{2\pi i}{k_\parallel}\frac{\Omega_\alpha}{k_\perp v_\perp}\left[\left(\frac{c}{B_0}\tilde{E}_z - \frac{\omega - k_\parallel v_\parallel}{k_\perp}\frac{\tilde{B}_x}{B_0}\right)(1-J_0^2) - iv_\perp J_0J_0'\frac{k_\parallel}{k_\perp}\frac{\tilde{B}_z}{B_0}\right]\left[k_\parallel\frac{\partial}{\partial v_\parallel} + 2(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp^2}\right]f_{\alpha 0}(v_\perp^2, v_\parallel). \tag{14}$$

The expressions for $\tilde{n}_\alpha$, $\tilde{j}_x$, and $\tilde{j}_y$ evaluated from Eqs. (12)–(14) provide adequate description of hydromagnetic waves in both low-β ($\beta \ll 1$) and high-β ($\beta > 1$) plasmas including finite gyroradius effects.

The expressions for $\tilde{n}_\alpha$, $\tilde{j}_x$, and $\tilde{j}_y$ that are obtained from Eqs. (12)–(14) (see Appendix A for some details) are

$$\frac{\tilde{n}_\alpha}{n_0} = \frac{2iq_\alpha}{m_\alpha k_\parallel \theta_{\alpha\perp}^2}\left\{\left[A_1(b_{\alpha\kappa}) + A_2(b_{\alpha\kappa})\frac{\theta_{\alpha\perp}^2}{2\theta_{\alpha\parallel}^2}Z_\kappa'(\xi_\alpha)\right]\tilde{E}_z - A_1(b_{\alpha\kappa})\frac{\omega}{ck_\perp}\tilde{B}_x\right\} + \left[A_3(b_{\alpha\kappa}) - A_4(b_{\alpha\kappa})\frac{\theta_{\alpha\perp}^2}{2\theta_{\alpha\parallel}^2}Z_\kappa'(\xi_\alpha)\right]\frac{\tilde{B}_z}{B_0}, \tag{15}$$

$$\tilde{j}_x = \sum_\alpha q_\alpha n_0\frac{k_\perp}{k_\parallel}\left[A_3(b_{\alpha\kappa}) - A_4(b_{\alpha\kappa})\frac{\theta_{\alpha\perp}^2}{2\theta_{\alpha\parallel}^2}Z_\kappa'(\xi_\alpha)\right]\frac{c}{B_0}\tilde{E}_z - \sum_\alpha q_\alpha n_0 A_3(b_{\alpha\kappa})\frac{\omega\tilde{B}_x}{k_\parallel B_0}$$
$$-i\sum_\alpha q_\alpha n_\alpha\frac{\Omega_\alpha}{k_\perp}\left[\left(\frac{\omega^2}{\Omega_\alpha^2} + \frac{\kappa_\parallel}{2\kappa_\parallel - 1}\frac{k_\parallel^2\theta_{\alpha\parallel}^2}{\Omega_\alpha^2}\right)A_5(b_{\alpha\kappa}) + \frac{k_\parallel^2\theta_{\alpha\perp}^2}{2\Omega_\alpha^2}A_6(b_{\alpha\kappa}) - A_7(b_{\alpha\kappa}) + A_8(b_{\alpha\kappa})\frac{\theta_{\alpha\perp}^2}{2\theta_{\alpha\parallel}^2}Z_\kappa'(\xi_\alpha)\right]\frac{\tilde{B}_z}{B_0}, \tag{16}$$

$$\tilde{j}_y = \sum_\alpha \frac{2iq_\alpha^2 n_0}{m_\alpha k_\| \theta_{\alpha\perp}^2}\frac{\omega}{k_\perp}A_1(b_{\alpha\kappa})\tilde{E}_z - i\sum_\alpha \frac{q_\alpha n_0}{b_{\alpha\kappa}}\frac{\Omega_\alpha}{k_\|}\left\{\frac{\omega^2}{\Omega_\alpha^2}A_1(b_{\alpha\kappa}) + \frac{k_\|^2\theta_{\alpha\perp}^2}{2\Omega_\alpha^2}\left[1 - A_2(b_{\alpha\kappa}) + \frac{2\kappa_\|}{2\kappa_\| - 1}\frac{\theta_{\alpha\|}^2}{\theta_{\alpha\perp}^2}A_1(b_{\alpha\kappa})\right]\right\}\frac{\tilde{B}_x}{B_0} + \sum_\alpha q_\alpha n_0 A_3(b_{\alpha\kappa})\frac{\omega}{k_\perp}\frac{\tilde{B}_z}{B_0}. \tag{17}$$

Here, $\xi_\alpha = \omega/(k_\| \theta_{\alpha\|})$, $Z'_\kappa(\xi_\alpha) = dZ_\kappa(\xi_\alpha)/d\xi_\alpha$, $b_{\alpha\kappa} = k_\perp^2 \theta_{\alpha\perp}^2/(2\Omega_\alpha^2)$, and the A_j are defined by

$$A_1(b_{\alpha\kappa}) = 2\int_0^\infty dv_\perp v_\perp (1-J_0^2)\frac{\partial}{\partial v_\perp^2}\frac{1}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong -\left(1 - \frac{3}{4}\frac{\kappa_\perp}{\kappa_\perp - 1}b_{\alpha\kappa}\right)b_{\alpha\kappa}, \quad \kappa_\perp > 1, \tag{18}$$

$$A_2(b_{\alpha\kappa}) \equiv \frac{2}{\theta_{\alpha\perp}^2}\int_0^\infty dv_\perp v_\perp \frac{J_0^2}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong \left[1 - \frac{\kappa_\perp}{\kappa_\perp - 1}b_{\alpha\kappa} + \frac{3}{4}\frac{\kappa_\perp^2}{(\kappa_\perp - 1)(\kappa_\perp - 2)}b_{\alpha\kappa}^2\right], \quad \kappa_\perp > 2, \tag{19}$$

$$A_3(b_{\alpha\kappa}) \equiv \frac{4\Omega_\alpha}{k_\perp \theta_{\alpha\perp}^2}\int_0^\infty dv_\perp v_\perp^2 J_0 J'_0 \frac{\partial}{\partial v_\perp^2}\frac{1}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong 1 - \frac{3}{2}\frac{\kappa_\perp}{\kappa_\perp - 1}b_{\alpha\kappa}, \quad \kappa_\perp > 1, \tag{20}$$

$$A_4(b_{\alpha\kappa}) \equiv \frac{4\Omega_\alpha}{k_\perp \theta_{\alpha\perp}^4}\int_0^\infty dv_\perp v_\perp^2 \frac{J_0 J'_0}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong -\frac{\kappa_\perp}{\kappa_\perp - 1}\left(1 - \frac{3}{2}\frac{\kappa_\perp}{\kappa_\perp - 2}b_{\alpha\kappa}\right), \quad \kappa_\perp > 2, \tag{21}$$

$$A_5(b_{\alpha\kappa}) \equiv \frac{8}{\theta_{\alpha\perp}^2}\int_0^\infty dv_\perp v_\perp^3 (J'_1)^2 \frac{\partial}{\partial v_\perp^2}\frac{1}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong -\left(1 - \frac{3\kappa_\perp}{\kappa_\perp - 1}b_{\alpha\kappa}\right), \quad \kappa_\perp > 1, \tag{22}$$

$$A_6(b_{\alpha\kappa}) \equiv \frac{8}{\theta_{\alpha\perp}^4}\int_0^\infty dv_\perp v_\perp^3 \frac{(J'_1)^2}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong \frac{\kappa_\perp}{\kappa_\perp - 1}\left(1 - \frac{3\kappa_\perp}{\kappa_\perp - 2}b_{\alpha\kappa}\right), \quad \kappa_\perp > 2, \tag{23}$$

$$A_7(b_{\alpha\kappa}) \equiv \frac{4}{\theta_{\alpha\perp}^2}\int_0^\infty dv_\perp v_\perp^3 (J'_0)^2 \frac{\partial}{\partial v_\perp^2}\frac{1}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong -\frac{2\kappa_\perp}{\kappa_\perp - 1}\left(1 - \frac{3}{2}\frac{\kappa_\perp}{\kappa_\perp - 2}b_{\alpha\kappa}\right)b_{\alpha\kappa}, \quad \kappa_\perp > 2, \tag{24}$$

$$A_8(b_{\alpha\kappa}) \equiv \frac{4}{\theta_{\alpha\perp}^4}\int_0^\infty dv_\perp v_\perp^3 \frac{(J'_0)^2}{(1+v_\perp^2/\kappa_\perp \theta_{\alpha\perp}^2)^{\kappa_\perp+1}} \cong \frac{2\kappa_\perp^2}{(\kappa_\perp - 1)(\kappa_\perp - 2)}\left(1 - \frac{3}{2}\frac{\kappa_\perp}{\kappa_\perp - 3}b_{\alpha\kappa}\right)b_{\alpha\kappa}, \quad \kappa_\perp > 3 \tag{25}$$

for $b_{\alpha\kappa} < 1$ (see Appendix A for the evaluation of the integrals). In the limit as $\kappa_\perp \to \infty$, $b_{\alpha\kappa} \to k_\perp^2 T_{\alpha\perp}/(m_\alpha \Omega_\alpha^2) \equiv b_\alpha$, which is a notation that is commonly used in the study of Maxwellian plasma, and A_j go over to the corresponding expressions for Maxwellian distribution. For example, in the limit as $\kappa_\perp \to \infty$, $A_1(b_{\alpha\kappa}) \to -[1 - I_0(b_\alpha)\exp(-b_\alpha)]$ and $A_2(b_{\alpha\kappa}) \to I_0(b_\alpha)\exp(-b_\alpha)$, where I_0 is the modified Bessel function. The function $Z_\kappa(\xi_\alpha)$ is the plasma dispersion function when the parallel velocity distribution is given by Eq. (4) and is defined by[16]

$$Z_\kappa(\xi_\alpha) = \frac{\Gamma(\kappa_\| + 1)}{\pi^{1/2}\kappa_\|^{1/2}\Gamma(\kappa_\| + 1/2)}\int_{-\infty}^{+\infty}\frac{ds}{(s - \xi_\alpha)(1 + s^2/\kappa_\|)^{(\kappa_\|+1)}}, \quad \mathrm{Im}\,\xi_\alpha > 0 \tag{26}$$

and by the analytic continuation of Eq. (26) for $\mathrm{Im}\,\xi_\alpha \le 0$. This is similar to $Z^*_\kappa(\xi)$ of Summers and Thorne.[2] The series and the asymptotic expansions of Z_κ for integer values of $\kappa_\|$ are[16]

$$Z_\kappa(\xi_\alpha) = i\sqrt{\pi}f(\kappa_\|)\left(1 - \frac{\kappa_\| + 1}{\kappa_\|}\xi_\alpha^2 + \cdots\right) - \frac{2\kappa_\| + 1}{\kappa_\|}\xi_\alpha\left(1 - \frac{2\kappa_\| + 3}{3\kappa_\|}\xi_\alpha^2 + \cdots\right), \quad |\xi_\alpha| \ll 1, \tag{27}$$

$$Z_\kappa(\xi_\alpha) = \frac{i\sqrt{\pi}f(\kappa_\|)}{(1 + \xi_\alpha^2/\kappa_\|)^{\kappa_\|+1}} - \frac{1}{\xi_\alpha}\left(1 + \frac{\kappa_\|}{2\kappa_\| - 1}\frac{1}{\xi_\alpha^2} + \cdots\right), \quad |\xi_\alpha| \gg 1, \tag{28}$$

where $f(\kappa_\|) \equiv \Gamma(\kappa_\| + 1)/[\kappa_\|^{1/2}\Gamma(\kappa_\| + 1/2)]$. For noninteger (including half-integer) values of $\kappa_\|$, the asymptotic expansion has correction terms,[18,19] which add small corrections to the real frequency of the excited waves. We ignore such corrections in the present analysis. The linear dispersion relation is obtained by substituting Eqs. (15)–(17) into Eqs. (1)–(3) and demanding nontrivial solutions of $\tilde{E}_z$, $\tilde{B}_x$, and $\tilde{B}_z$.

III. ANALYSIS OF HYDROMAGNETIC WAVES

A. Shear Alfven waves

In the study of shear Alfven waves, compressional component of the magnetic perturbation ($\tilde{B}_z$) may be ignored, while the magnetic field line bending represented by $\tilde{B}_x$ is more important. This simplifies the analysis considerably. We assume $b_{e\kappa}\approx 0$ but retain finite ion gyroradius ($b_{i\kappa}<1$) effects. After substituting the appropriate expressions for $\tilde{n}_\alpha$ and $\tilde{j}_y$ into Eqs. (1) and (3), we obtain

$$\left\{Z'_\kappa(\xi_e)+\frac{2m_e\theta_{e\parallel}^2}{m_i\theta_{i\perp}^2}\left[A_1(b_{i\kappa})+A_2(b_{i\kappa})\frac{\theta_{i\perp}^2}{2\theta_{i\parallel}^2}Z'_\kappa(\xi_i)\right]\right\}\tilde{E}_z=\frac{2m_e\theta_{e\parallel}^2}{m_i\theta_{i\perp}^2}\frac{\omega}{ck_\perp}A_1(b_{i\kappa})\tilde{B}_x, \tag{29}$$

$$\left[1+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{\omega^2}{k_\parallel^2V_A^2}+g(b_{i\kappa},\beta)\right]\tilde{B}_x=\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{\omega ck_\perp}{k_\parallel^2V_A^2}\tilde{E}_z, \tag{30}$$

where

$$g(b_{i\kappa},\beta)=\frac{1}{2}(\beta_{e\perp}-\beta_{e\parallel})+\frac{1}{2}\left[\left(\frac{\kappa_\perp-1}{\kappa_\perp}\right)\frac{1-A_2(b_{i\kappa})}{b_{i\kappa}}\beta_{i\perp}+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\beta_{i\parallel}\right]. \tag{31}$$

Here electron terms that are of the order of m_e/m_i in comparison with the ion terms have been neglected. Also, we have used the definitions of $b_{i\kappa}$ and $V_A^2=B_0^2/(4\pi m_in_0)$, where V_A is the Alfven speed. By combining Eq. (29) with Eq. (30) we then obtain the kinetic dispersion relation for shear Alfven waves, which can be written as

$$\left[1+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{\omega^2}{k_\parallel^2V_A^2}+g(b_{i\kappa},\beta)\right]\left[Z'_\kappa(\xi_e)+\frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}A_2(b_{i\kappa})Z'_\kappa(\xi_i)\right]+\frac{m_e}{m_i}\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{k_\perp^2\theta_{e\parallel}^2}{\Omega_i^2}[1+g(b_{i\kappa},\beta)]=0. \tag{32}$$

We now solve Eq. (32) for different limiting values of $|\xi_\alpha|$. For convenience, we shall use the following definitions in our presentation:

$$L_\kappa(\xi_\alpha)=\sqrt{\pi}f(\kappa_\parallel)[(\kappa_\parallel+1)/\kappa_\parallel]\xi_\alpha,\quad |\xi_\alpha|\ll 1, \tag{33}$$

$$\tilde{L}_\kappa(\xi_\alpha)=\sqrt{\pi}f(\kappa_\parallel)[(\kappa_\parallel+1)/\kappa_\parallel]\xi_\alpha/(1+\xi_\alpha^2/\kappa_\parallel)^{\kappa_\parallel+2},\quad |\xi_\alpha|\gg 1. \tag{34}$$

First, we consider $|\xi_i|\gg 1$ and $|\xi_e|\gg 1$, i.e., $\theta_{i\parallel},\theta_{e\parallel}\ll\omega/k_\parallel$ (cold electrons and cold ions). Referring to Eq. (28), we realize that the term $(m_e/m_i)(\theta_{e\parallel}^2/\theta_{i\parallel}^2)A_2(b_{i\kappa})Z'_\kappa(\xi_i)$ in Eq. (32) may be neglected in comparison with $Z'_\kappa(\xi_e)$ as $m_e/m_i\ll 1$ and $|\xi_i|\gg|\xi_e|$ typically. Using the leading terms in the asymptotic expansion of $Z'_\kappa(\xi_e)$, we then find

$$\left[1+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{\omega^2}{k_\parallel^2V_A^2}+g(b_{i\kappa},\beta)\right][1-2i\tilde{L}_\kappa(\xi_e)\xi_e^2]+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{c^2k_\perp^2}{\omega_{pe}^2}\frac{\omega^2}{k_\parallel^2V_A^2}[1+g(b_{i\kappa},\beta)]=0, \tag{35}$$

where $\omega_{pe}^2=4\pi e^2n_0/m_e$. Without the imaginary term that accounts for wave-particle interaction, Eq. (35) yields

$$\frac{\omega^2}{k_\parallel^2V_A^2}=-\frac{b_{i\kappa}}{A_1(b_{i\kappa})}\frac{1+g(b_{i\kappa},\beta)}{1+(c^2k_\perp^2/\omega_{pe}^2)[1+g(b_{i\kappa},\beta)]} \tag{36}$$

Since $|\xi_\alpha|\sim V_A/\theta_{\alpha\parallel}\gg 1$ requires $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{e\parallel}\ll m_e/m_i\ll 1$, $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{i\parallel}\ll 1$ and since we can assume $\beta_{\alpha\perp}\sim\beta_{\alpha\parallel}$ for typical plasma, the dispersion relation (36) is valid for low-$\beta(\beta\ll 1)$ plasma. For such plasma we may take $g\ll 1$. Then, Eq. (36) becomes

$$\frac{\omega^2}{k_\parallel^2V_A^2}=-\frac{b_{i\kappa}}{A_1(b_{i\kappa})}\frac{1}{1+(c^2k_\perp^2/\omega_{pe}^2)}. \tag{37}$$

When $\kappa_\perp\to\infty$, Eq. (37) becomes identical to the dispersion relation for the inertial shear Alfven wave that was derived earlier for Maxwellian plasma.[20–22] So, Eq. (37) is the dispersion relation for inertial shear Alfven wave in kappa distribution plasma. The inertial shear Alfven wave has been suggested[20,21] as a mechanism for particle acceleration just above the auroral ionosphere where the plasma is cold enough so that $\beta_{e\parallel}\ll m_e/m_i$. We may note that without the $(c^2k_\perp^2/\omega_{pe}^2)$ term, the dispersion relation follows directly from Eq. (30) if we set $\tilde{E}_z$ equal to zero (MHD limit). The $(c^2k_\perp^2/\omega_{pe}^2)$ term arises from the non-MHD feature ($\tilde{E}_z\neq 0$) and denotes the kinetic modification due to finite electron skin depth (c/ω_{pe}). It may also be verified that $b_{i\kappa}\ll c^2k_\perp^2/\omega_{pe}^2$ for the low-β plasma defined above. Since $A_1/b_{i\kappa}\cong -1$ and $(1-A_2)/b_{i\kappa}\cong\kappa_\perp/(\kappa_\perp-1)$ when $b_{i\kappa}\ll 1$, Eq. (37) further reduces to

$$\frac{\omega^2}{k_\parallel^2V_A^2}=\frac{1}{1+(c^2k_\perp^2/\omega_{pe}^2)}, \tag{38}$$

which is identical to the dispersion relation for Maxwellian plasma.[20–22] This is expected because under the conditions $|\xi_\alpha|\gg 1$, $b_{\alpha\kappa}\approx 0$ and with the omission of the wave-particle interaction, details of the velocity distribution do not matter and so whether the distribution is kappa or Maxwellian is unimportant. With the inclusion of the imaginary term, which is small compared to unity, the approximate dispersion relation for low-β plasma with $b_{i\kappa}\approx 0$ is

$$\frac{\omega^2}{k_\parallel^2V_A^2}=\frac{1}{1+c^2k_\perp^2/\omega_{pe}^2}\left[1-2i\frac{c^2k_\perp^2/\omega_{pe}^2}{1+c^2k_\perp^2/\omega_{pe}^2}\tilde{L}_\kappa(\xi_e)\xi_e^2\right]. \tag{39}$$

It exhibits electron Landau damping of the inertial shear Alfven wave in kappa distribution plasma.

Second, we consider $|\xi_i|\gg 1$ and $|\xi_e|\ll 1$, i.e., $\theta_{i\parallel}\ll\omega/k_\parallel\ll\theta_{e\parallel}$ (hot electrons and cold ions). For simplicity of presentation we ignore the terms proportional to $Z'_\kappa(\xi_i)$. This eliminates the ion sound mode, which is of no interest here. Using the leading terms of the series expansion for $Z'_\kappa(\xi_e)$ in Eq. (32) we then find

$$\left[1+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{\omega^2}{k_\parallel^2V_A^2}+g(b_{i\kappa},\beta)\right]\left[1+i\frac{2\kappa_\parallel}{2\kappa_\parallel+1}L_\kappa(\xi_e)\right]-\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}\frac{A_1(b_{i\kappa})}{b_{i\kappa}}k_\perp^2\rho_s^2[1+g(b_{i\kappa},\beta)]=0. \tag{40}$$

Here $\rho_s=c_s/\Omega_i$ and $c_s^2=p_{e\parallel}/(m_in_0)=[\kappa_\parallel/(2\kappa_\parallel-1)](m_e/m_i)\theta_{e\parallel}^2\equiv T_{e\parallel}/m_i$. Without the imaginary term, Eq. (40) yields

$$\frac{\omega^2}{k_\parallel^2V_A^2}=\left[-\frac{b_{i\kappa}}{A_1(b_{i\kappa})}+\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}k_\perp^2\rho_s^2\right][1+g(b_{i\kappa},\beta)]. \tag{41}$$

The condition $|\xi_e|\sim V_A/\theta_{e\parallel}\ll 1$ requires $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{e\parallel}\gg m_e/m_i$ and, as before, $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{i\parallel}\ll 1$. Because of these conditions we may neglect $\beta_{i\parallel}$ and $\beta_{i\perp}$ (considering $\beta_{i\perp}\sim\beta_{i\parallel}$ in typical plasma) in g and rewrite Eq. (41) as

$$\frac{\omega^2}{k_\parallel^2V_A^2}=\left[-\frac{b_{i\kappa}}{A_1(b_{i\kappa})}+\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}k_\perp^2\rho_s^2\right]\left[1+\frac{1}{2}(\beta_{e\perp}-\beta_{e\parallel})\right]$$
$$\cong\left[1+\frac{\kappa_\perp}{\kappa_\perp-1}\left(\frac{3}{4}+\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}\frac{T_{e\parallel}}{T_{i\perp}}\right)b_{i\kappa}\right]\times\left[1+\frac{1}{2}(\beta_{e\perp}-\beta_{e\parallel})\right]. \tag{42}$$

In the last step of Eq. (42) we have used the small-$b_{i\kappa}$ expansion of $A_1(b_{i\kappa})$ [refer to Eq. (18)] and the relation $k_\perp^2\rho_s^2=[\kappa_\perp/(\kappa_\perp-1)](T_{e\parallel}/T_{i\perp})b_{i\kappa}$.

If $(m_e/m_i)[2\kappa_\parallel/(2\kappa_\parallel-1)]\ll\beta_{e\parallel}\ll 1$ and $\beta_{e\perp}\sim\beta_{e\parallel}\ll 1$, then Eq. (42) describes the kinetic shear Alfven wave in low-β kappa distribution plasma consisting of hot electrons and cold ions. Unlike the ideal MHD shear Alfven wave, the kinetic shear Alfven wave has $\tilde{E}_z(\neq 0)$ associated with it and incorporates finite ion gyroradius effect. In the limit as $\kappa_\parallel,\kappa_\perp\to\infty$, we recover the dispersion relation for the kinetic Alfven wave in Maxwellian plasma,[23,24] which has been invoked for plasma heating[23] and for auroral particle acceleration.[24] With the imaginary term in Eq. (40) included, the approximate dispersion relation is

$$\frac{\omega^2}{k_\parallel^2V_A^2}\cong-\frac{b_{i\kappa}}{A_1(b_{i\kappa})}+\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}k_\perp^2\rho_s^2\left[1-i\frac{2\kappa_\parallel}{2\kappa_\parallel+1}L_\kappa(\xi_e)\right]. \tag{43}$$

It describes electron Landau damping of the kinetic Alfven wave in low-β kappa distribution plasma.

For high-β_e ($\beta_e>1$) plasma, on the other hand, Eq. (42) describes nonoscillatory, purely growing modes when

$$\beta_{e\parallel}-\beta_{e\perp}>2. \tag{44}$$

The instability condition is the same as that for the fire-hose (garden-hose) instability[25–27] in Maxwellian plasma with anisotropic electron pressures ($\beta_{e\parallel}>\beta_{e\perp}$). The growth rate of the instability is different for kappa distribution plasma if the finite ion gyroradius effect is retained.

Next, we consider the reverse situation $|\xi_i|\ll 1$, $|\xi_e|\gg 1$, i.e., $\theta_{e\parallel}\ll\omega/k_\parallel\ll\theta_{i\parallel}$ (cold electrons and hot ions). Using the series expansion of $Z'_\kappa(\xi_i)$ and the asymptotic expansion of $Z'_\kappa(\xi_e)$ and keeping only the leading terms in Eq. (32), we find

$$\left[1+\frac{A_1(b_{i\kappa})}{b_{i\kappa}}\frac{\omega^2}{k_\parallel^2V_A^2}+g(b_{i\kappa},\beta)\right]\times\left\{1+\frac{i}{A_2(b_{i\kappa})}\frac{2\kappa_\parallel}{2\kappa_\parallel+1}\left[\frac{m_i\theta_{i\parallel}^2}{m_e\theta_{e\parallel}^2}\tilde{L}_\kappa(\xi_e)+A_2(b_{i\kappa})L_\kappa(\xi_i)\right]\right\}-\frac{2\kappa_\parallel}{2\kappa_\parallel+1}\frac{\theta_{i\parallel}^2}{\theta_{i\perp}^2}\frac{A_1(b_{i\kappa})}{A_2(b_{i\kappa})}[1+g(b_{i\kappa},\beta)]=0. \tag{45}$$

Without the imaginary term, Eq. (45) yields

$$\frac{\omega^2}{k_\parallel^2V_A^2}=-\frac{b_{i\kappa}}{A_1(b_{i\kappa})}\left[1-\frac{2\kappa_\parallel}{2\kappa_\parallel+1}\frac{\theta_{i\parallel}^2}{\theta_{i\perp}^2}\frac{A_1(b_{i\kappa})}{A_2(b_{i\kappa})}\right][1+g(b_{i\kappa},\beta)]. \tag{46}$$

The dispersion relation (46) is valid for $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{e\parallel}\ll m_e/m_i$ and $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{i\parallel}\gg 1$. If we further use the small-$b_{i\kappa}$ expansion of A_1 and A_2, then Eq. (46) becomes

$$\frac{\omega^2}{k_\parallel^2V_A^2}\cong\left[1+\frac{\kappa_\perp}{\kappa_\perp-1}\left(\frac{3}{4}+\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}\frac{T_{i\parallel}}{T_{i\perp}}\right)b_{i\kappa}\right]\times\left[1+\frac{1}{2}(\beta_{i\perp}-\beta_{i\parallel})\right]. \tag{47}$$

Comparison with Eq. (42) indicates that the role of the electron temperatures is now played by the ion temperatures. When $\beta_{i\parallel}-\beta_{i\perp}<2$, Eq. (47) describes another type of kinetic shear Alfven wave in kappa distribution plasma consisting of cold electrons and hot ions. On the other hand, if $\beta_{i\parallel}-\beta_{i\perp}>2$, Eq. (47) describes nonoscillatory, purely growing modes, which are driven unstable by the ion pressure anisotropy. This is similar to the fire-hose instability[25–27] condition [Eq. (44)]. The imaginary term in Eq. (45) represents the Landau damping (both electron and ion) of the modes.

Finally, we consider $|\xi_i|\ll 1$ and $|\xi_e|\ll 1$, i.e., $\theta_{i\parallel},\theta_{e\parallel}\gg\omega/k_\parallel$ (hot electrons and hot ions). Using the leading terms of the series expansion for $Z'_\kappa(\xi_\alpha)$ in Eq. (32), we obtain the dispersion relation

$$\frac{\omega^2}{k_\parallel^2V_A^2}\left\{1+\frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}A_2+i\frac{2\kappa_\parallel}{2\kappa_\parallel+1}\left[L_\kappa(\xi_e)+\frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}A_2L_\kappa(\xi_i)\right]\right\}\frac{A_1}{b_{i\kappa}}+\left\{1+\frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}\left(A_2-\frac{2\kappa_\parallel}{2\kappa_\parallel+1}\frac{\theta_{i\parallel}^2}{\theta_{i\perp}^2}A_1\right)+i\frac{2\kappa_\parallel}{2\kappa_\parallel+1}\left[L_\kappa(\xi_e)+\frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}A_2L_\kappa(\xi_i)\right]\right\}(1+g)=0. \tag{48}$$

When $b_{i\kappa}\ll 1$, Eq. (48) reduces to

$$\frac{\omega^2}{k_\parallel^2V_A^2}=\left[1+\frac{1}{2}(\beta_\perp-\beta_\parallel)\right], \tag{49}$$

where $\beta_\perp=\Sigma_\alpha\beta_{\alpha\perp}$ and $\beta_\parallel=\Sigma_\alpha\beta_{\alpha\parallel}$. The dispersion relation (49) applies to high-β plasma since $|\xi_\alpha|\sim V_A/\theta_{\alpha\parallel}\ll 1$ requires $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{e\parallel}\gg m_e/m_i$ and $[(2\kappa_\parallel-1)/2\kappa_\parallel]\beta_{i\parallel}\gg 1$. It indicates instability due to pressure anisotropy when $\beta_\parallel-\beta_\perp>2$. The instability condition is similar to Eq. (44) except that both electron and ion pressures play roles.

B. Compressional Alfven waves

For the compressional Alfven waves, we may neglect $\tilde{B}_x$ in comparison with $\tilde{B}_z$, which simplifies the analysis. We also consider long perpendicular wavelength modes ($b_{\alpha\kappa} \ll 1$). Keeping the leading terms in $A_1(b_{\alpha\kappa})-A_8(b_{\alpha\kappa})$ and substituting the resulting expressions for $\tilde{n}_\alpha$ and $\tilde{j}_x$ into Eqs. (1) and (2), we find

$$\left[Z'_\kappa(\xi_e) + \frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}Z'_\kappa(\xi_i)\right]\frac{ie\tilde{E}_z}{m_e k_\parallel \theta_{e\parallel}^2} = \frac{\kappa_\perp}{\kappa_\perp - 1}\left[\frac{\theta_{e\perp}^2}{2\theta_{e\parallel}^2}Z'_\kappa(\xi_e) - \frac{\theta_{i\perp}^2}{2\theta_{i\parallel}^2}Z'_\kappa(\xi_i)\right]\frac{\tilde{B}_z}{B_0}, \quad (50)$$

$$\left\{1 - \frac{\omega^2}{k^2V_A^2} + \frac{k_\parallel^2}{2k^2}\sum_\alpha(\beta_{\alpha\perp} - \beta_{\alpha\parallel}) + \frac{k_\perp^2}{k^2}\sum_\alpha \beta_{\alpha\perp}\left[1 + \frac{\kappa_\perp}{\kappa_\perp - 2}\frac{\theta_{\alpha\perp}^2}{2\theta_{\alpha\parallel}^2}Z'_\kappa(\xi_\alpha)\right]\right\}\tilde{B}_z = -i\frac{k_\perp^2}{2k^2}\frac{\kappa_\perp}{\kappa_\perp - 1}\sum_\alpha \frac{4\pi q_\alpha n_0}{k_\parallel B_0}\frac{\theta_{\alpha\perp}^2}{\theta_{\alpha\parallel}^2}Z'_\kappa(\xi_\alpha)\tilde{E}_z, \quad (51)$$

where electron terms that are of the order of m_e/m_i in comparison with the ion terms have been neglected and $k_\perp^2\rho_s^2 \ll 1$ has been assumed. It is important to note that Eqs. (50) and (51) are valid for $\kappa_\parallel > 1/2$ and $\kappa_\perp > 2$. As before, we study the stability properties of the modes for different limiting values of $|\xi_\alpha|$.

First, we consider $|\xi_i| \gg 1$ and $|\xi_e| \gg 1$, i.e., $\theta_{i\parallel}, \theta_{e\parallel} \ll \omega/k_\parallel$ (cold electrons and cold ions). Using the asymptotic expansions of $Z'_\kappa(\xi_\alpha)$, keeping only the leading terms by noting that $m_e/m_i \ll 1$ and $|\xi_i| \gg |\xi_e|$ typically, and then combining Eqs. (50) and (51) we derive the dispersion relation

$$1 - \frac{\omega^2}{k^2V_A^2} + (\beta_\perp - \beta_\parallel)\frac{k_\parallel^2}{2k^2} + \beta_\perp\frac{k_\perp^2}{k^2} - \beta_{e\perp}^2\frac{m_i}{4m_e}\frac{k_\perp^2}{k^2}\frac{k_\parallel^2V_A^2}{\omega^2} - i\frac{\kappa_\parallel}{2\kappa_\parallel - 1}\frac{\kappa_\perp}{\kappa_\perp - 2}\frac{\beta_{e\perp}^2}{\beta_{e\parallel}}\frac{k_\perp^2}{k^2}\tilde{L}_\kappa(\xi_e) = 0. \quad (52)$$

A possible root of the quartic equation, which is consistent with $|\xi_e| \gg 1$, is obtained when $\beta_{e\perp}^2(m_i/m_e)(k_\perp^2/4k^2) \times(k_\parallel^2V_A^2/\omega^2)$ term in the equation can be neglected. This, in essence, amounts to neglecting the term associated with $\tilde{E}_z$ on the right-hand side of Eq. (51) and the condition for it is $\beta_{e\perp}^2 \ll (m_e/m_i)[k^4/(k_\parallel^2k_\perp^2)]$. When the condition is satisfied, the dispersion relation is

$$1 - \frac{\omega^2}{k^2V_A^2} + (\beta_\perp - \beta_\parallel)\frac{k_\parallel^2}{2k^2} + \beta_\perp\frac{k_\perp^2}{k^2} - i\frac{\kappa_\parallel}{2\kappa_\parallel - 1}\frac{\kappa_\perp}{\kappa_\perp - 2}\frac{\beta_{e\perp}^2}{\beta_{e\parallel}}\frac{k_\perp^2}{k^2}\tilde{L}_\kappa(\xi_e) = 0, \quad (53)$$

which describes the damped compressional Alfven mode in kappa distribution plasma including finite pressure effects. In the cold plasma limit ($\theta_{\alpha\parallel}, \theta_{\alpha\perp} \to 0$), it reduces to $\omega = kV_A$. The real part of Eq. (53) is the same as that in Maxwellian plasma.[1] The imaginary term represents electron transit-time damping (magnetic analogue of Landau damping) in kappa distribution plasma.

In the parameter regimes $|\xi_i| \gg 1$ and $|\xi_e| \ll 1$, i.e., $\theta_{i\parallel} \ll \omega/k_\parallel \ll \theta_{e\parallel}$ (hot electrons and cold ions), using the leading terms of the appropriate expansions in Eqs. (50) and (51), and neglecting $k_\parallel^2\theta_{i\parallel}^2/\omega^2$ compared to terms of the order of unity or larger, we find

$$\left\{1 - \frac{2\kappa_\parallel - 1}{2\kappa_\parallel + 1}\frac{k_\parallel^2c_s^2}{\omega^2} + i\frac{2\kappa_\parallel}{2\kappa_\parallel + 1}\left[L_\kappa(\xi_e) + \frac{\beta_{e\parallel}}{\beta_{i\parallel}}\tilde{L}_\kappa(\xi_i)\right]\right\}\frac{ie\tilde{E}_z}{m_e k_\parallel \theta_{e\parallel}^2} = \frac{\kappa_\parallel}{2\kappa_\parallel - 1}\frac{\beta_{e\perp}}{\beta_{e\parallel}}\left\{1 + i\frac{2\kappa_\parallel}{2\kappa_\parallel + 1}\left[L_\kappa(\xi_e) - \frac{\beta_{e\parallel}}{\beta_{e\perp}}\frac{\beta_{i\perp}}{\beta_{i\parallel}}\tilde{L}_\kappa(\xi_i)\right]\right\} \times\frac{\tilde{B}_z}{B_0}, \quad (54)$$

$$\left\{1 - \frac{\omega^2}{k^2V_A^2} + \frac{k_\parallel^2}{2k^2}(\beta_\perp - \beta_\parallel) + \frac{k_\perp^2}{k^2}\left(\beta_\perp - \frac{\kappa_\perp - 1}{\kappa_\perp - 2}\frac{2\kappa_\parallel + 1}{2\kappa_\parallel - 1}\frac{\beta_{e\perp}^2}{\beta_{e\parallel}}\right) - i\frac{\kappa_\perp - 1}{\kappa_\perp - 2}\frac{2\kappa_\parallel}{2\kappa_\parallel - 1}\frac{k_\perp^2}{k^2}\left[\frac{\beta_{e\perp}^2}{\beta_{e\parallel}}L_\kappa(\xi_e) + \frac{\beta_{i\perp}^2}{\beta_{i\parallel}}\tilde{L}_\kappa(\xi_i)\right]\right\}\frac{\tilde{B}_z}{B_0} = -\beta_{e\perp}\frac{k_\perp^2}{k^2}\left\{\frac{2\kappa_\parallel + 1}{2\kappa_\parallel} + i\left(L_\kappa(\xi_e) - \frac{\beta_{e\parallel}}{\beta_{e\perp}}\frac{\beta_{i\perp}}{\beta_{i\parallel}}\tilde{L}_\kappa(\xi_i)\right)\right\} \times\frac{ie\tilde{E}_z}{m_e k_\parallel \theta_{e\parallel}^2}. \quad (55)$$

By neglecting the imaginary terms in Eqs. (54) and (55), we obtain the dispersion relation

$$\left(1 - \frac{2\kappa_\parallel - 1}{2\kappa_\parallel + 1}\frac{k_\parallel^2c_s^2}{\omega^2}\right)\left[1 - \frac{\omega^2}{k^2V_A^2} + \frac{k_\parallel^2}{2k^2}(\beta_\perp - \beta_\parallel) + \frac{k_\perp^2}{k^2}\left(\beta_\perp - \frac{\kappa_\perp - 1}{\kappa_\perp - 2}\frac{2\kappa_\parallel + 1}{2\kappa_\parallel - 1}\frac{\beta_{e\perp}^2}{\beta_{e\parallel}}\right)\right] + \frac{k_\perp^2}{2k^2}\frac{2\kappa_\parallel + 1}{2\kappa_\parallel - 1}\frac{\beta_{e\perp}^2}{\beta_{e\parallel}} = 0. \quad (56)$$

When $k_\parallel^2/k^2 \ll 2/\beta_\parallel$ and $\beta_\parallel > 1$, one of the roots of Eq. (56) corresponds to the "slow" magnetosonic mode with $\omega^2 \sim[(2\kappa_\parallel - 1)/(2\kappa_\parallel + 1)]k_\parallel^2c_s^2$, and the root is approximately given by

$$\frac{\omega^2}{k_\parallel^2c_s^2}\left[1 + \beta_\perp - \frac{\beta_{e\perp}^2}{2\beta_{e\parallel}}\frac{\kappa_\perp}{\kappa_\perp - 2}\frac{2\kappa_\parallel + 1}{2\kappa_\parallel - 1}\right] = \frac{2\kappa_\parallel - 1}{2\kappa_\parallel + 1}\left(1 + \beta_\perp - \frac{\beta_{e\perp}^2}{\beta_{e\parallel}}\frac{\kappa_\perp - 1}{\kappa_\perp - 2}\frac{2\kappa_\parallel + 1}{2\kappa_\parallel - 1}\right). \quad (57)$$

The mode becomes unstable if the inequality

$$\frac{\kappa_\perp - 2}{\kappa_\perp - 1} < \frac{2\kappa_\parallel + 1}{2\kappa_\parallel - 1}\frac{\beta_{e\perp}}{1 + \beta_\perp}\frac{\beta_{e\perp}}{\beta_{e\parallel}} < \frac{2(\kappa_\perp - 2)}{\kappa_\perp} \quad (58)$$

is satisfied. The other root corresponds to the "fast" magnetosonic mode $\omega \sim kV_A$ and is approximately given by

$$\frac{\omega^2}{k^2V_A^2} \cong 1+\beta_\perp - \frac{\beta_{e\perp}^2}{2\beta_{e\parallel}}\frac{\kappa_\perp}{\kappa_\perp-2}\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}. \tag{59}$$

The instability condition for this mode is

$$\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\frac{\beta_{e\perp}}{1+\beta_\perp}\frac{\beta_{e\perp}}{\beta_{e\parallel}} > \frac{2(\kappa_\perp-2)}{\kappa_\perp}. \tag{60}$$

In the limit as $\kappa_\parallel, \kappa_\perp \to \infty$, instability conditions (58) and (60) reduce to the ones that were found for Maxwellian plasma, and the instabilities were referred to as the field-swelling instabilities.[28,29] For some finite values of $\kappa_\parallel(>1/2)$ and $\kappa_\perp(>2)$, the range of values of $\beta_{e\perp}$ for instability, as defined by inequality (58), can be much narrower and the threshold value of $\beta_{e\perp}(>\beta_{e\parallel})$ for instability, as suggested by inequality (60), can be much lower than those for Maxwellian plasma. In the case of weak pressure anisotropy so that the plasma is near marginal stability, the imaginary terms in Eqs. (54) and (55), which take into account Landau damping and transit-time damping by electrons and ions, should be retained in the analysis.

Next, we consider the reverse situation, i.e., $|\xi_i| \ll 1$, $|\xi_e| \gg 1$ (cold electrons and hot ions), and assume that $\theta_e \approx 0$. In this case, Eq. (50) yields

$$\frac{ie\tilde{E}_z}{m_ik_\parallel\theta_{i\perp}^2} \cong \frac{\kappa_\perp}{\kappa_\perp-1}\frac{2\kappa_\parallel+1}{2\kappa_\parallel}\frac{m_e}{m_i}\frac{\omega^2}{k_\parallel^2\theta_{i\parallel}^2}\frac{\tilde{B}_z}{B_0} \ll \frac{\tilde{B}_z}{B_0}. \tag{61}$$

Therefore, we may set the right-hand side of Eq. (51) equal to zero. Moreover, we observe that $\omega^2/(k^2V_A^2) \ll (k_\parallel^2/k^2)(\theta_{i\parallel}^2/V_A^2)=[(2\kappa_\parallel-1)/2\kappa_\parallel](k_\parallel^2/k^2)\beta_{i\parallel}$ as $|\xi_i| \ll 1$. Then the dispersion relation that follows from Eq. (51) is

$$1+\frac{k_\parallel^2}{2k^2}(\beta_{i\perp}-\beta_{i\parallel})+\frac{k_\perp^2}{k^2}\beta_{i\perp} \times\left\{1-\frac{\kappa_\perp-1}{\kappa_\perp-2}\frac{2\kappa_\parallel}{2\kappa_\parallel-1}\frac{\beta_{i\perp}}{\beta_{i\parallel}}\left[\frac{2\kappa_\parallel+1}{2\kappa_\parallel}+iL_\kappa(\xi_i)\right]\right\}=0. \tag{62}$$

Here we have used $\theta_{i\perp}^2/\theta_{i\parallel}^2=[(\kappa_\perp-1)/\kappa_\perp][2\kappa_\parallel/(2\kappa_\parallel-1)] \times(\beta_{i\perp}/\beta_{i\parallel})$. Referring to the definition of $L_\kappa(\xi_i)$ we solve Eq. (62) for ω and find

$$\omega=-i\frac{k_\parallel\theta_{i\parallel}}{2\sqrt{\pi}f(\kappa_\parallel)}\frac{\kappa_\perp-2}{\kappa_\perp-1}\frac{2\kappa_\parallel-1}{\kappa_\parallel+1}\frac{\beta_{i\parallel}}{\beta_{i\perp}^2}\frac{k^2}{k_\perp^2} \times\left[\frac{k_\parallel^2}{k^2}\left(1+\frac{\beta_{i\perp}-\beta_{i\parallel}}{2}\right) +\frac{k_\perp^2}{k^2}\left(1+\beta_{i\perp}-\frac{\kappa_\perp-1}{\kappa_\perp-2}\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\frac{\beta_{i\perp}^2}{\beta_{i\parallel}}\right)\right]. \tag{63}$$

Instability (Im $\omega>0$) occurs when

$$\frac{k_\parallel^2}{k^2}\left(1+\frac{\beta_{i\perp}-\beta_{i\parallel}}{2}\right)+\frac{k_\perp^2}{k^2}\left(1+\beta_{i\perp}-\frac{\kappa_\perp-1}{\kappa_\perp-2}\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\frac{\beta_{i\perp}^2}{\beta_{i\parallel}}\right)<0. \tag{64}$$

For nearly parallel propagation ($k_\parallel/k_\perp \gg 1$), the instability condition becomes $\beta_{i\parallel}-\beta_{i\perp}>2$, which is same as the condition for the fire-hose (garden-hose) instability of shear Alfven wave due to ion pressure anisotropy [see Eq. (47) and the discussion following it] and which is the same for both kappa and Maxwellian plasmas. On the other hand, for nearly perpendicular propagation ($k_\parallel/k_\perp \ll 1$), the instability condition is

$$\frac{\beta_{i\perp}^2}{\beta_{i\parallel}} > \frac{\kappa_\perp-2}{\kappa_\perp-1}\frac{2\kappa_\parallel-1}{2\kappa_\parallel+1}(1+\beta_{i\perp}), \tag{65}$$

which is the condition for the mirror instability in kappa distribution plasma. In the limit as $\kappa_\parallel, \kappa_\perp \to \infty$, we recover the instability condition for the mirror instability in Maxwellian plasma.[30] For some finite values of $\kappa_\parallel(>1/2)$ and $\kappa_\perp(>2)$, the factor on the right-hand side of Eq. (65) can be significantly smaller than unity and thus the threshold value of $\beta_{i\perp}^2/\beta_{i\parallel}$ for mirror instability can be significantly lower in kappa distribution plasma.

Finally, we consider $|\xi_i| \ll 1$ and $|\xi_e| \ll 1$, i.e., $\theta_{i\parallel}, \theta_{e\parallel} \gg \omega/k_\parallel$ (hot electrons and hot ions). If the imaginary terms in the series expansion of $Z_\kappa'(\xi_\alpha)$ are not included in Eqs. (50) and (51), we have

$$\left(1+\frac{m_e\theta_{e\parallel}^2}{m_i\theta_{i\parallel}^2}\right)\frac{ie\tilde{E}_z}{m_ek_\parallel\theta_{e\parallel}^2}=\frac{\kappa_\perp}{2(\kappa_\perp-1)}\left(\frac{\theta_{e\perp}^2}{\theta_{e\parallel}^2}-\frac{\theta_{i\perp}^2}{\theta_{i\parallel}^2}\right)\frac{\tilde{B}_z}{B_0}, \tag{66}$$

$$\left[1-\frac{\omega^2}{k^2V_A^2}+\frac{k_\parallel^2}{2k^2}(\beta_\perp-\beta_\parallel) +\frac{k_\perp^2}{k^2}\sum_\alpha\beta_{\alpha\perp}\left(1-\frac{\kappa_\perp}{\kappa_\perp-2}\frac{2\kappa_\parallel+1}{2\kappa_\parallel}\frac{\theta_{\alpha\perp}^2}{\theta_{\alpha\parallel}^2}\right)\right]\frac{\tilde{B}_z}{B_0} =-i\frac{\kappa_\perp}{\kappa_\perp-1}\frac{2\kappa_\parallel+1}{2\kappa_\parallel}\frac{k_\perp^2}{k^2}\frac{4\pi en_0}{k_\parallel B_0^2}\left(\frac{\theta_{e\perp}^2}{\theta_{e\parallel}^2}-\frac{\theta_{i\perp}^2}{\theta_{i\parallel}^2}\right)\tilde{E}_z, \tag{67}$$

respectively. For the convenient case of $\theta_{e\perp}^2/\theta_{e\parallel}^2=\theta_{i\perp}^2/\theta_{i\parallel}^2$ so that $\tilde{E}_z=0$, the dispersion relation that follows from Eq. (67) is

$$\frac{\omega^2}{k^2V_A^2}=\left[1+\frac{k_\parallel^2}{2k^2}(\beta_\perp-\beta_\parallel) +\frac{k_\perp^2}{k^2}\sum_\alpha\beta_{\alpha\perp}\left(1-\frac{\kappa_\perp-1}{\kappa_\perp-2}\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\frac{\beta_{\alpha\perp}}{\beta_{\alpha\parallel}}\right)\right] \tag{68}$$

when $\theta_{\alpha\perp}^2/\theta_{\alpha\parallel}^2$ is related to $\beta_{\alpha\perp}/\beta_{\alpha\parallel}$. The instability condition can be written as

$$\frac{k_\parallel^2}{k^2}\left(1+\frac{\beta_\perp-\beta_\parallel}{2}\right) +\frac{k_\perp^2}{k^2}\left[1+\sum_\alpha\beta_{\alpha\perp}\left(1-\frac{\kappa_\perp-1}{\kappa_\perp-2}\frac{2\kappa_\parallel+1}{2\kappa_\parallel-1}\frac{\beta_{\alpha\perp}}{\beta_{\alpha\parallel}}\right)\right]<0. \tag{69}$$

This is similar to Eq. (64) and so the discussion following Eq. (64) applies, except that in this case both the hot electrons and the hot ions participate in the instability process. In particular, for nearly perpendicular propagation ($k_\parallel/k_\perp \ll 1$) and for $\beta_{e\perp}/\beta_{e\parallel}=\beta_{i\perp}/\beta_{i\parallel}$, the instability condition is

$$\beta_{\perp}\frac{\beta_{i\perp}}{\beta_{i\parallel}} > \frac{\kappa_{\perp}-2}{\kappa_{\perp}-1}\frac{2\kappa_{\parallel}-1}{2\kappa_{\parallel}+1}(1+\beta_{\perp}), \tag{70}$$

where $\beta_{\perp}=\Sigma_{\alpha}\beta_{\alpha\perp}$. The instability condition (70) is evidently different from the corresponding condition for Maxwellian plasma, which is obtained from Eq. (70) by taking the limits $\kappa_{\parallel},\kappa_{\perp}\to\infty$. When the imaginary term in the series expansion of $Z'_{\kappa}(\xi_{\alpha})$ is included, the dispersion relation is modified as

$$\frac{\omega^2}{k^2V_A^2} = \left\{ 1 + \frac{k_{\parallel}^2}{2k^2}(\beta_{\perp}-\beta_{\parallel}) + \frac{k_{\perp}^2}{k^2}\sum_{\alpha}\beta_{\alpha\perp} \times\left[1 - \frac{\kappa_{\perp}-1}{\kappa_{\perp}-2}\frac{2\kappa_{\parallel}+1}{2\kappa_{\parallel}-1}\frac{\beta_{\alpha\perp}}{\beta_{\alpha\parallel}}\left(1+i\frac{2\kappa_{\parallel}}{2\kappa_{\parallel}+1}L_{\kappa}(\xi_{\alpha})\right)\right]\right\}, \tag{71}$$

and it takes into account the electron and the ion transit-time damping.

For all the cases considered above, the Landau damping and the transit-time damping rates of the hydromagnetic waves in kappa distribution plasma can be significantly different from those in Maxwellian plasma depending on the allowed values of $\kappa_{\parallel}(>1/2)$ and $\kappa_{\perp}(>2)$. Further discussion on it is presented in Sec. IV.

IV. DISCUSSION AND PHYSICAL INTERPRETATION

We have classified the hydromagnetic waves and their stability properties in kappa distribution plasma according to the relative magnitudes of the wave phase speed and the electron and ion thermal speeds ($|\xi_e|$ and $|\xi_i|$) and considered only the limiting cases for which solutions of the appropriate dispersion relations are analytically tractable. Such analytical approach is obviously limited in scope and applicability; but it helps to understand the important basic features of the waves. A more rigorous analysis requires numerical solution of the dispersion relations. Conclusions reached here for a specific choice of the kappa distribution function would remain qualitatively unchanged for other possible choices of the kappa distribution.[2,16]

The well-known results for the hydromagnetic waves in Maxwellian plasma are easily recovered from the results reported here by taking the limits $\kappa_{\parallel},\kappa_{\perp}\to\infty$. The comparison shows important differences between the kappa distribution plasma and the Maxwellian plasma. First, the magnitude of the resonant wave-particle interaction in the suprathermal region of the velocity space ($v_{\parallel}=\omega/k_{\parallel}\gg\theta_{\alpha\parallel}$) is considerably larger in kappa distribution plasma than in Maxwellian plasma. This is evident from Fig. 1 where the behavior of $-\mathrm{Im}\,Z'$ as a function of $|\omega/(k_{\parallel}\theta_{\parallel})|$ is shown for two types of velocity distribution. The reason for the marked difference in the magnitudes of $-\mathrm{Im}\,Z'$ (which is proportional to $\partial f_0/\partial v_{\parallel}$ at $v_{\parallel}=\omega/k_{\parallel}$) when $|\omega/(k_{\parallel}\theta_{\parallel})|\gg 1$ is that the slope of the kappa distribution in $v_{\parallel}$-space decreases according to a power law, whereas the slope decreases exponentially in the case of the Maxwellian distribution. Hence, both the Landau damping and the transit-time damping (magnetic analogue of Landau damping) of the waves are enhanced and, consequently, the threshold values for the excitation of unstable hydromagnetic waves in kappa distribution plasma are increased. Other important differences are the following. In the parameter regimes $|\xi_i|\gg 1$ and $|\xi_e|\ll 1$, (1) the frequency of the oscillatory stable mode (e.g., kinetic shear Alfven wave) in kappa distribution plasma is different from that in Maxwellian plasma [see Eq. (42)] and (2) the instability conditions for the nonoscillatory, purely growing compressional modes, which are excited due to pressure anisotropy, are different from those in Maxwellian plasma [see Eqs. (58) and (60)]. The condition for the mirror instability, which is excited in the parameter regimes $|\xi_i|\ll 1$ and $|\xi_e|\gg 1$, is also different [see Eq. (65)]. The differences can be attributed to the fact that the density and temperature perturbations of the charged particle species α in kappa distribution plasma are different from those in Maxwellian plasma when $|\xi_{\alpha}|\ll 1$. This interesting aspect is discussed below.

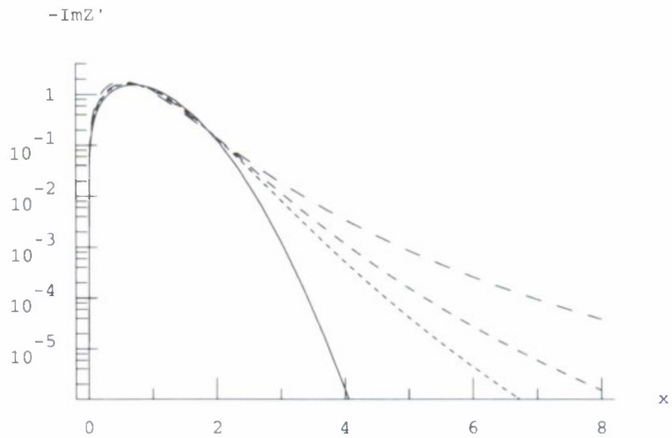

FIG. 1. Comparison of $-\mathrm{Im}\,Z'$ vs x, where $x=|\omega/(k_{\parallel}\theta_{\parallel})|$. Solid curve is for Maxwellian distribution. Dashed curves with decreasing lengths of segments are for kappa distributions with $\kappa_{\parallel}=2$, 4, and 6, respectively.

When $b_{\alpha\kappa}\approx 0$ and $|\xi_{\alpha}|\ll 1$, neglecting the imaginary term and keeping the leading term in the rest of the series expansion of $Z'_{\kappa}(\xi_{\alpha})$ in Eq. (15), we find

$$\frac{\tilde{n}_{\alpha}}{n_0} = -\frac{iq_{\alpha}}{k_{\parallel}T_{\alpha\parallel}}\frac{2\kappa_{\parallel}+1}{2\kappa_{\parallel}-1}\tilde{E}_z + \left(1-\frac{2\kappa_{\parallel}+1}{2\kappa_{\parallel}-1}\frac{T_{\alpha\perp}}{T_{\alpha\parallel}}\right)\frac{\tilde{B}_z}{B_0} \tag{72}$$

after relating $\theta_{\alpha\parallel}^2$ and $\theta_{\alpha\perp}^2$ to $T_{\alpha\parallel}$ and $T_{\alpha\perp}$. This is evidently different from the corresponding expression for Maxwellian plasma, which is given by the $\kappa_{\parallel}\to\infty$ limit of Eq. (72). Similarly, by calculating the perturbed parallel pressure $\tilde{p}_{\alpha\parallel}$ from Eq. (12) we find

$$\frac{\tilde{p}_{\alpha\parallel}}{p_{\alpha\parallel}} = -\frac{iq_{\alpha}}{k_{\parallel}T_{\alpha\parallel}}\tilde{E}_z + \left(1-\frac{T_{\alpha\perp}}{T_{\alpha\parallel}}\right)\frac{\tilde{B}_z}{B_0}, \tag{73}$$

which is clearly the same for both kappa and Maxwellian plasmas. That $\tilde{p}_{\alpha\parallel}/p_{\alpha\parallel}$ should be the same for any choice of equilibrium distribution is easily understood if we recognize that it also follows from the parallel (to $\mathbf{B}_0$) component of the linearized momentum balance equation[28,29]

$$0 = -ik_{\parallel}\tilde{p}_{\alpha\parallel} + q_{\alpha}n_0\tilde{E}_z + ik_{\parallel}(p_{\alpha\parallel}-p_{\alpha\perp})(\tilde{B}_z/B_0). \tag{74}$$

Equation (74) is derived from the linearized Vlasov equation by taking the appropriate velocity moment and by neglecting the inertia term for the considered low frequency waves. We

then notice from Eqs. (72) and (73) that $\tilde{p}_{\alpha\|}/p_{\alpha\|} \neq \tilde{n}_\alpha/n_0$. This suggests a nonzero temperature perturbation $\tilde{T}_{\alpha\|}$ in kappa distribution plasma given by

$$\frac{\tilde{T}_{\alpha\|}}{T_{\alpha\|}} = \frac{2}{2\kappa_\| - 1}\left(\frac{iq_\alpha \tilde{E}_z}{k_\| T_{\alpha\|}} + \frac{T_{\alpha\perp}}{T_{\alpha\|}}\frac{\tilde{B}_z}{B_0}\right). \tag{75}$$

In contrast, $\tilde{p}_{\alpha\|}/p_{\alpha\|}=\tilde{n}_\alpha/n_0$ and, hence, $\tilde{T}_{\alpha\|}=0$ in Maxwellian plasma under the same low frequency conditions.[28,29] That $\tilde{T}_{\alpha\|}=0$ in Maxwellian plasma also follows from Eq. (75) by taking the limit $\kappa_\| \to \infty$.

We next discuss $\tilde{p}_{\alpha\perp}/p_{\alpha\perp}$, which plays role in determining the excitation conditions of the field swelling and mirror instabilities [see Eqs. (60) and (65)]. Calculating $\tilde{p}_{\alpha\perp}/p_{\alpha\perp}$ from Eq. (12) and keeping the leading order terms when $|\xi_\alpha| \ll 1$ and $b_{\alpha\kappa} \approx 0$, we find

$$\frac{\tilde{p}_{\alpha\perp}}{p_{\alpha\perp}} \cong -\frac{iq_\alpha}{k_\| T_{\alpha\|}}\frac{2\kappa_\|+1}{2\kappa_\|-1}\tilde{E}_z + 2\left(1 - \frac{\kappa_\perp - 1}{\kappa_\perp - 2}\frac{2\kappa_\|+1}{2\kappa_\|-1}\frac{T_{\alpha\perp}}{T_{\alpha\|}}\right)\frac{\tilde{B}_z}{B_0}. \tag{76}$$

Subtracting $\tilde{n}_\alpha/n_0$ we also find

$$\frac{\tilde{T}_{\alpha\perp}}{T_{\alpha\perp}} \cong \left[1 - \frac{\kappa_\perp}{\kappa_\perp - 2}\frac{2\kappa_\|+1}{2\kappa_\|-1}\frac{T_{\alpha\perp}}{T_{\alpha\|}}\right]\frac{\tilde{B}_z}{B_0}. \tag{77}$$

The corresponding expressions for Maxwellian plasma[28,29] are obtained by taking the $\kappa_\|, \kappa_\perp \to \infty$ limits, and the differences between the kappa distribution plasma and the Maxwellian plasma are evident. Quantitative estimates of the differences depend on the specific values of $\kappa_\| (>1/2)$ and $\kappa_\perp (>2)$.

In Appendix B, we present a more convenient form of the perturbed distribution function $\tilde{f}_\alpha$ that may be used instead of Eq. (12) to obtain $\tilde{n}_\alpha$, $\tilde{p}_{\alpha\|}$, and $\tilde{p}_{\alpha\perp}$ given by Eqs. (72), (73), and (76).

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APPENDIX A: DERIVATION OF EQS. (15)–(17)

In obtaining $\tilde{n}_\alpha$, $\tilde{j}_x$, and $\tilde{j}_y$ from Eqs. (12)–(14), we need to evaluate a set of integrals in $(v_\|, v_\perp)$ space. When Eq. (4) is used for $f_{\alpha 0}$, the needed integrals in $v_\|$ space are

$$\int_{-\infty}^{+\infty} dv_\| v_\|^l g_{\alpha 0} \quad \text{and} \quad \int_{-\infty}^{+\infty} \frac{dv_\|}{\omega - k_\| v_\|}\frac{\partial g_{\alpha 0}}{\partial v_\|}, \tag{A1}$$

where $l=0,2$ and $g_{\alpha 0}(v_\|) \equiv [1+v_\|^2/(\kappa_\| \theta_{\alpha\|}^2)]^{-(\kappa_\|+1)}$. Other integrals in $v_\|$ space are either zero due to symmetry or can be reduced to the first integral after partial integrations. The first integral is evaluated with the change in integration variable as

$$\begin{aligned}\int_{-\infty}^{+\infty} dv_\| v_\|^l g_{\alpha 0} &= (\kappa_\|^{1/2}\theta_{\alpha\|})^{l+1}\int_0^\infty dt \frac{t^{(l-1)/2}}{(1+t)^{\kappa_\|+1}} \\ &= (\kappa_\|^{1/2}\theta_{\alpha\|})^{l+1}\frac{\Gamma[(l+1)/2]\Gamma[\kappa_\| - (l-1)/2]}{\Gamma(\kappa_\|+1)}\end{aligned} \tag{A2}$$

for $\kappa_\| > (l-1)/2$, where in the last step we have used the standard integral[31]

$$\int_0^\infty dt \frac{t^{a-1}}{(1+t)^{a+b}} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \quad (\mathrm{Re}\, a>0,\ \mathrm{Re}\, b>0). \tag{A3}$$

The second integral after partial integration and change in integration variable is

$$\begin{aligned}\int_{-\infty}^{+\infty} \frac{dv_\|}{\omega - k_\| v_\|}\frac{\partial g_{\alpha 0}}{\partial v_\|} &= k_\| \frac{\partial}{\partial \omega}\int_{-\infty}^{+\infty} dv_\| \frac{g_{\alpha 0}}{(\omega - k_\| v_\|)} \\ &= -\frac{1}{k_\| \theta_{\alpha\|}}\frac{\partial}{\partial \xi_\alpha}\int_{-\infty}^{+\infty}\frac{ds}{(s-\xi_\alpha)(1+s^2/\kappa_\|)^{\kappa_\|+1}},\end{aligned} \tag{A4}$$

where $\xi_\alpha = \omega/(k_\| \theta_{\alpha\|})$. It is related to $Z_\kappa'(\xi_\alpha) = dZ_\kappa(\xi_\alpha)/d\xi_\alpha$, where $Z_\kappa(\xi_\alpha)$ is defined by Eq. (26). The needed integrals in $v_\perp$ space are

$$\begin{aligned}&\int_0^\infty dv_\perp v_\perp J_0^2 h_{\alpha 0}, \quad \int_0^\infty dv_\perp v_\perp (1-J_0^2)\frac{\partial h_{\alpha 0}}{\partial v_\perp^2}, \\ &\int_0^\infty dv_\perp v_\perp^2 J_0 J_0' h_{\alpha 0}, \quad \int_0^\infty dv_\perp v_\perp^2 J_0 J_0' \frac{\partial h_{\alpha 0}}{\partial v_\perp^2}, \\ &\int_0^\infty dv_\perp v_\perp^3 (J_0')^2 h_{\alpha 0}, \quad \int_0^\infty dv_\perp v_\perp^3 (J_0')^2 \frac{\partial h_{\alpha 0}}{\partial v_\perp^2}, \\ &\int_0^\infty dv_\perp v_\perp^3 (J_1')^2 h_{\alpha 0}, \quad \int_0^\infty dv_\perp v_\perp^3 (J_1')^2 \frac{\partial h_{\alpha 0}}{\partial v_\perp^2},\end{aligned} \tag{A5}$$

where $h_{\alpha 0}(v_\perp^2) \equiv [1+v_\perp^2/(\kappa_\perp \theta_{\alpha\perp}^2)]^{-(\kappa_\perp+1)}$. When the series expansions of J_0 and J_1 are used and the leading few terms are kept, the above integrals are of the form $\int_0^\infty dv_\perp v_\perp^l [1+v_\perp^2/(\kappa_\perp \theta_{\alpha\perp}^2)]^{-(\kappa_\perp+p)}$, where $l=1,3,5,\ldots$ and $p=1,2$. It is evaluated by changing the integration variable and then using the standard integral [Eq. (A3)] and is given by

$$\begin{aligned}&\int_0^\infty dv_\perp v_\perp^l [1+v_\perp^2/(\kappa_\perp \theta_{\alpha\perp}^2)]^{-(\kappa_\perp+p)} \\ &= (\kappa_\perp^{1/2}\theta_{\alpha\perp})^{l+1}\Gamma[(l+1)/2] \\ &\quad \times \Gamma[\kappa_\perp + p - (l+1)/2]/[2\Gamma(\kappa_\perp + p)]\end{aligned} \tag{A6}$$

for $\kappa_\perp > [(l+1)/2]-p$. Evaluating the leading terms of the integrals in Eq. (A5) using Eq. (A6), we obtain the A_j given by Eqs. (18)–(25). The expressions for $\tilde{n}_\alpha$, $\tilde{j}_x$, and $\tilde{j}_y$ given by Eqs. (15)–(17) are obtained when the above results are used.

APPENDIX B: A MORE CONVENIENT FORM OF $\tilde{f}_\alpha$ WHEN $b_{\alpha\kappa}\approx 0$ AND $|\omega/(k_\|\theta_{\alpha\|})|\ll 1$

Here we construct the portion of the perturbed distribution function $\tilde{f}_\alpha$ that is most relevant to the calculation of $\tilde{n}_\alpha$, $\tilde{p}_{\alpha\|}$, and $\tilde{p}_{\alpha\perp}$ when $b_{\alpha\kappa}\approx 0$ and $|\omega/(k_\|\theta_{\alpha\|})|\ll 1$. (Landau damping and transit-time damping are considered unimportant.) Since $|\omega/\Omega_\alpha|\ll 1$, we argue that f_α will be a function of the magnetic moment $\mu=m_\alpha v_\perp^2/(2B)$, which is a valid adiabatic invariant, and the total particle energy in the wave field ε. Keeping in mind that the equilibrium distribution function is a kappa distribution of the form given by Eq. (4), we choose

$$f_\alpha(\mu,\varepsilon)=C\left(1+\frac{2\mu B_0}{\kappa_\perp m_\alpha\theta_{\alpha\perp}^2}\right)^{-(\kappa_\perp+1)}\left[1+\frac{2(\varepsilon-\mu B_0)}{\kappa_\| m_\alpha\theta_{\alpha\|}^2}\right]^{-(\kappa_\|+1)}, \tag{B1}$$

where C is a normalization constant. Using $\mu\cong[m_\alpha v_\perp^2/(2B_0)](1-\tilde{B}_z/B_0)\equiv\mu_0(1-\tilde{B}_z/B_0)$, $\varepsilon=\varepsilon_0+q_\alpha\tilde{\phi}$, where $\varepsilon_0=m_\alpha(v_\perp^2+v_\|^2)/2$ and $\tilde{\phi}$ is the electrostatic potential, and expanding f_α in a Taylor series around $\mu=\mu_0$ and $\varepsilon=\varepsilon_0$ we obtain

$$\begin{aligned}f_\alpha(\mu,\varepsilon)\cong{}& f_{\alpha0}-\frac{2(\kappa_\|+1)}{\kappa_\|}\left[1+\frac{2}{\kappa_\| m_\alpha\theta_{\alpha\|}^2}(\varepsilon_0-\mu_0B_0)\right]^{-1}\frac{q_\alpha\tilde{\phi}}{m_\alpha\theta_{\alpha\|}^2}f_{\alpha0}\\&+\frac{2\mu_0B_0}{m_\alpha\theta_{\alpha\perp}^2}\left\{\frac{\kappa_\perp+1}{\kappa_\perp}\left(1+\frac{2\mu_0B_0}{\kappa_\perp m_\alpha\theta_{\alpha\perp}^2}\right)^{-1}-\frac{\kappa_\|+1}{\kappa_\|}\frac{\theta_{\alpha\perp}^2}{\theta_{\alpha\|}^2}\left[1+\frac{2}{\kappa_\| m_\alpha\theta_{\alpha\|}^2}(\varepsilon_0-\mu_0B_0)\right]^{-1}\right\}\frac{\tilde{B}_z}{B_0}f_{\alpha0}.\end{aligned} \tag{B2}$$

Here $f_{\alpha0}=f_\alpha(\mu=\mu_0,\varepsilon=\varepsilon_0)$ and the additional terms represent the perturbed distribution function $\tilde{f}_\alpha$. Using $\tilde{\phi}\cong i\tilde{E}_z/k_\|$ for the considered low frequency waves and returning to the velocity space variables, we find

$$\tilde{f}_\alpha(v_\perp^2,v_\|)=-\left\{\frac{2(\kappa_\|+1)}{\kappa_\|}\frac{1}{1+v_\|^2/(\kappa_\|\theta_{\alpha\|}^2)}\frac{iq_\alpha\tilde{E}_z}{m_\alpha k_\|\theta_{\alpha\|}^2}-\frac{v_\perp^2}{\theta_{\alpha\perp}^2}\left[\frac{\kappa_\perp+1}{\kappa_\perp}\frac{1}{1+v_\perp^2/(\kappa_\perp\theta_{\alpha\perp}^2)}-\frac{\kappa_\|+1}{\kappa_\|}\frac{\theta_{\alpha\perp}^2}{\theta_{\alpha\|}^2}\frac{1}{1+v_\|^2/(\kappa_\|\theta_{\alpha\|}^2)}\right]\frac{\tilde{B}_z}{B_0}\right\}f_{\alpha0}, \tag{B3}$$

and $f_{\alpha0}(v_\perp^2,v_\|)$ is same as that given by Eq. (4). With this convenient form of $\tilde{f}_\alpha$, the necessary velocity integrations can be carried out more easily by using Eq. (A3), and it may be verified that $\tilde{n}_\alpha$, $\tilde{p}_{\alpha\|}$, and $\tilde{p}_{\alpha\perp}$ calculated from Eq. (B3) are identical to those given in Sec. IV from a more complete form of $\tilde{f}_\alpha$.

[1]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962), pp. 196–204.
[2]D. Summers and R. M. Thorne, Phys. Fluids B **3**, 1835 (1991) and references therein.
[3]V. M. Vasyliunas, J. Geophys. Res., [Space Phys.] **73**, 2839, DOI:10.1029/JA073i009p02839 (1968).
[4]R. M. Thorne and D. Summers, Phys. Fluids B **3**, 2117 (1991).
[5]D. Summers and R. M. Thorne, J. Geophys. Res., [Space Phys.] **97**, 16827, DOI:10.1029/92JA01664 (1992).
[6]Z. Meng, R. M. Thorne, and D. Summers, J. Plasma Phys. **47**, 445 (1992).
[7]S. Xue, R. M. Thorne, and D. Summers, J. Geophys. Res., [Space Phys.] **98**, 17475, DOI:10.1029/93JA00790 (1993).
[8]D. Summers, S. Xue, and R. M. Thorne, Phys. Plasmas **1**, 2012 (1994).
[9]D. Summers and R. M. Thorne, J. Plasma Phys. **53**, 293 (1995).
[10]M. A. Hellberg and R. L. Mace, Phys. Plasmas **9**, 1495 (2002).
[11]R. L. Mace and M. A. Hellberg, Phys. Plasmas **10**, 21 (2003).
[12]R. L. Mace, Phys. Plasmas **10**, 2181 (2003).
[13]R. L. Mace, Phys. Plasmas **11**, 507 (2004).
[14]R. P. Singhal and A. K. Tripathi, Phys. Plasmas **13**, 012102 (2006).
[15]F. Xiao, Q. Zhou, H. He, H. Zheng, and S. Wang, J. Geophys. Res., [Space Phys.] **112**, A07219, DOI:10.1029/2006JA012050 (2007).
[16]B. Basu, Phys. Plasmas **15**, 042108 (2008).
[17]N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973), pp. 402–405.
[18]R. L. Mace and M. A. Hellberg, Phys. Plasmas **2**, 2098 (1995).
[19]D. Summers, R. M. Thorne, and H. Matsumoto, Phys. Plasmas **3**, 2496 (1996).
[20]C. K. Goertz and R. W. Boswell, J. Geophys. Res., [Space Phys.] **84**, 7239, DOI:10.1029/JA084iA12p07239 (1979).
[21]C. K. Goertz, Planet. Space Sci. **32**, 1387 (1984).
[22]R. L. Lysak and W. Lotko, J. Geophys. Res., [Space Phys.] **101**, 5085, DOI:10.1029/95JA03712 (1996).
[23]A. Hasegawa and L. Chen, Phys. Rev. Lett. **35**, 370 (1975).
[24]A. Hasegawa, J. Geophys. Res., [Space Phys.] **81**, 5083, DOI:10.1029/JA081i028p05083 (1976).
[25]M. N. Rosenhluth, Los Alamos Laboratory Report No. LA-2030, 1956.
[26]S. Chandrasekhar, A. N. Kaufman, and K. M. Watson, Proc. R. Soc. London, Ser. A **245**, 435 (1958).
[27]A. A. Vedenov and R. Z. Sagdeev, Izv. Akad. Nauk. SSR, Ser. Khi. **3**, 278 (1958).
[28]B. Basu and B. Coppi, Phys. Rev. Lett. **48**, 799 (1982).
[29]B. Basu and B. Coppi, Phys. Fluids **27**, 1187 (1984).
[30]A. Hasegawa, Phys. Fluids **12**, 2642 (1969).
[31]P. J. Davis, in *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965), p. 258.